# Minisymposium 20

# Nonlinear and Stochastic Optimization

*Leiter des Symposiums:*


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## Donnerstag, 21. September
Hörsaal 411 AVZ I, Endenicher Allee 11-13

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15:00 – 15:50 **Stefan Volkwein** *(Graz)*
Proper orthogonal Decompostion (POD) for parametric PDEs and for optimaliy systems

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16:00 – 16:20 **Roland Griesse** *(RICAM Linz)*
Elliptic Optimal Control Problems with Mixed Constraints

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16:30 – 16:50 **Arnd Rösch** *(RICAM Linz)*
On optimal control problems with mixed control-state constraints

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17:00 – 17:20 **Ilia Gherman** *(Trier)*
Efficient Methods for Aerodynamic Optimization

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17:30 – 17:50 **Harald Held** *(Duisburg-Essen)*
Shape Optimization Under Uncertainty – A Stochastic Programming Perspective

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## Freitag, 22. September
Hörsaal 411 AVZ I, Endenicher Allee 11-13

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15:00 – 15:50 **Werner Römisch** *(HU Berlin)*
Multistage stochastic programs: Stability and scenario trees

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16:00 – 16:50 **Marc C. Steinbach** *(Zuse Institute Berlin, FH Vorarlberg)*
Optimal Control of Gas and Water Networks

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17:00 – 17:50 **Oliver Stein** *(Aachen)*
Adaptive convexification for robust optimization problems

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# Vortragsauszüge

**Stefan Volkwein** *(Graz)*
Proper orthogonal Decompostion (POD) for parametric PDEs and for optimaliy systems

POD is a powerful technique for model reduction of linear and non-linear systems. It is based on a Galerkin type discretization with basis elements created from the dynamical system itself. First, POD is used to derive low-order models for a so-called $\lambda$-$\omega$-system that is a universal model to investigate two-species reaction-diffusion problems. In the case of fast reaction kinetics and small diffusion, these systems evolve to turbulent behavior. The performance of the POD model reduction is studied in dependence on the parameters of the $\lambda$-$\omega$-system. With increasing turbulence more POD modes are needed to capture the dynamics of the full system in a satisfactory way. Secondly, POD is applied to estimate parameters in elliptic partial differential equations. The parameter estimation is formulated in terms of an optimal control problem that is solved by an augmented Lagrangian method combined with a sequential quadratic programming algorithm. In the context of optimal control this approach may suffer from the fact that the basis elements are computed from a reference trajectory containing features which are quite different from those of the optimally controlled trajectory. Finally, a method is proposed which avoids this problem of unmodelled dynamics in the proper orthogonal decomposition approach to optimal control. It is referred to as optimality system proper orthogonal decomposition (OS-POD). The results are joined work with M. Kahlbacher, K. Kunisch, and H. Müller at the University of Graz.

**Roland Griesse** *(RICAM Linz)*
Elliptic Optimal Control Problems with Mixed Constraints

In this talk we consider the following class of linear-quadratic optimal control problems with state $y$ and control $u$:

$$(\mathbf{P}(\delta)) \qquad \text{Minimize } \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u - u_d\|^2_{L^2(\Omega)} - \int_\Omega y\,\delta_1\,dx - \int_\Omega u\,\delta_2\,dx$$

subject to $u \in L^2(\Omega)$ and the elliptic state equation

$$(3) \qquad \begin{aligned} -\Delta y &= u + \delta_3 \quad \text{on } \Omega \\ y &= 0 \quad \text{on } \partial\Omega \end{aligned}$$

as well as pointwise pure and mixed control-state constraints

$$(4) \qquad \begin{aligned} u - \delta_4 &\geq 0 \quad \text{on } \Omega \\ \varepsilon u + y - \delta_5 &\geq y_c \quad \text{on } \Omega. \end{aligned}$$

Problem $(\mathbf{P}(\delta))$ depends on a parameter $\delta = (\delta_1, \delta_2, \delta_3, \delta_4, \delta_5)$, and we prove the Lipschitz stability of the unique optimal solution in $L^\infty(\Omega)$, with respect to perturbations in $\delta$. The presence of simultaneous control and mixed constraints (4) requires a refinement of previously used techniques.

**Arnd Rösch** *(RICAM Linz)*
### On optimal control problems with mixed control-state constraints

Optimal control problems with mixed control-state constraints exhibit a lot of the positive properties of control constrained problems: Lagrange multiplier are measurable and bounded, optimal solutions are Lipschitz continuous for distributed elliptic control problems. Moreover, such problems can approximate optimal control problems with pure state constraints. If control constraints are given in addition, then the approximation error can be estimated. Moreover, we will show new results concerning the discretization error for this class of problems.

**Ilia Gherman** *(Trier)*
### Efficient Methods for Aerodynamic Optimization

Constructing a new optimization method for geometric design of an aircraft is a very challenging task. It is clear that the optimization should be based on the existing flow solvers. A further demand on the optimization method is that it should have a low relative computational complexity.

In the MEGADESIGN project, we developed a *one-shot* method which is based on (partially) reduced SQP methods. The idea is to solve all of the equations simultaneously.

The method uses existing flow solvers provided to us by the German Airspace Center. Within this framework, also additional state constraints can be included.

The developed one-shot method was efficiently applied to 2D and 3D drag minimization problems with and without additional state constraints. In this talk we will present the method as well as the numerical results.

**Harald Held** *(Duisburg-Essen)*
Shape Optimization Under Uncertainty – A Stochastic Programming Perspective

*Harald Held* (Department of Mathematics, University of Duisburg-Essen)
*Martin Rumpf* (Institute for Numerical Simulation, University of Bonn)
*Rüdiger Schultz* (Department of Mathematics, University of Duisburg-Essen)

We consider an elastic body subjected to internal and external forces which are uncertain. The deformations are described by PDEs that are solved efficiently by Composite Finite Elements. The objective is, for example, to minimize a least square error compared to a target displacement. A gradient method using the shape derivative together with a level-set method is employed to solve the problem.

We show that the structure of this problem is similar to that of a two-stage stochastic linear programming problem: In the first stage, the non-anticipative decision on the shape has to be taken. Afterwards, the realizations of the random forces are observed, and the variational formulation of the elasticity system takes the role of the second-stage problem.

**Werner Römisch** *(HU Berlin)*
Multistage stochastic programs: Stability and scenario trees

Multistage stochastic programs are regarded as optimization problems in spaces of integrable functions. The stability of such optimization problems with respect to perturbations of the stochastic input process is adressed. We review some recent stability results and discuss their use for designing scenario tree approximations of the stochastic input process. We present a general algorithmic framework for the generation of such tree approximations and report on its implementation. Numerical experience is provided for scenario tree approximations of multivariate processes in electricity portfolio management.

**Marc C. Steinbach** *(Zuse Institute Berlin, FH Vorarlberg)*
Optimal Control of Gas and Water Networks

Operative planning in supply networks with nonlinear fluid dynamics leads to large-scale discrete-continuous optimization problems over graphs. The lecture focuses on the structural analysis of such DAE and PDE network models under fixed combinatorial decisions. We present topological index criteria for DAE arising in the incompressible case and discuss implications of the index on boundary value problems resulting from full discretizations in space and time. Observing that the associated large, structured KKT systems permit a decoupling of space and time based on parallel spatial projections, we develop highly efficient solution algorithms using interior methods. Comments on the issue of discrete decisions (pump switching) are also provided. For the municipal water supply network of Berlin, we finally present results of minimum-cost operation under reliable demand forecast.

**Oliver Stein** *(Aachen)*
Adaptive convexification for robust optimization problems

We present a new numerical solution method for robust optimization problems in the absence of convexity. Its main idea is to adaptively construct convex relaxations of the lower level problem, replace the relaxed lower level problems equivalently by their Karush-Kuhn-Tucker conditions, and solve the resulting mathematical programs with complementarity constraints. In contrast to the commonly used approaches, this approximation produces *feasible iterates* for the original robust problem.

The convex relaxations are constructed with ideas from the $\alpha$BB method of global optimization. The necessary upper bounds for functions on box domains can be determined using the techniques of interval arithmetic, where our algorithm already works if only one such bound is available for the problem.

We show convergence of stationary points of the approximating problems to a stationary point of original robust problem within arbitrarily given tolerances. Numerical examples illustrate the performance of the method.