Optimal Control of Gas and Water Networks

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Overview

- Application Background
- Optimization Model
- MINLP
- Gas Management
- Summary



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Task

Operative Planning





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Operative Planning

Find: Operating schedule of network satisfying the predicted demand subject to physical, technical, contractual, and hygienic restrictions





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Operative Planning

Find: Operating schedule of network satisfying the predicted demand subject to physical, technical, contractual, and hygienic restrictions

Goal: Reduction of daily operating cost



Task

Operative Planning

40000 40000 Find: Operating schedule of network 35000 satisfying the predicted demand 30000 subject to physical, technical, contractual, and hygienic restrictions 25000 20000 -20000 Goal: Reduction of daily operating cost 15000 10000 10000 5000 5000 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 **Optimization Project** (Berlin) BWB – Berliner Wasserbetriebe (Mannheim) ABB Utilities GmbH (ZIB) Bernd Gnädig, M. Steinbach

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Hardware



Household connection \longrightarrow

 \leftarrow Drinking water network





Hardware

 \leftarrow



Household connection \longrightarrow

— Drinking water network



This is *not* Berlin, it is Tenerife!

Photographs by Mr. Bald (Stadtwerke Plettenberg) http://www.wasser.de

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Network Topology

Directed Graph $G = (\mathcal{N}, \mathcal{A})$

Node types

Arc types

 $\mathcal{A}_{\text{pi}} \ \text{pipe}$

 \mathcal{A}_{pu} pump \mathcal{A}_{vl} valve



Hydrodynamics

Planning Horizon

 $t \in I := [0, T]$ with T = 24 h; initial values at t = 0

Dynamic Variables

- Arcs: Volumetric flowrate $Q_{ij}(t)$
- Nodes: Pressure head $H_j(t) = h_j + p_j(t)/(g\rho)$



Hydrodynamics





Network Elements

Nodes

- Reservoir (source): linear
- Junction (demand): linear
- Tank (buffer): inflow $A_j(H_j, t)\dot{H}_j$





Network Elements

Nodes

- Reservoir (source): linear
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- Tank (buffer): inflow $A_j(H_j, t)\dot{H}_j$



Arcs

• Pipe: Hydraulic pressure loss $\Delta H_{ij}(Q_{ij}) \approx \phi_{ij}(Q_{ij})$



Network Elements

Nodes

- Reservoir (source): linear
- Junction (demand): linear
- Tank (buffer): inflow $A_j(H_j, t)\dot{H}_j$



• Pipe: Hydraulic pressure loss $\Delta H_{ij}(Q_{ij}) \approx \phi_{ij}(Q_{ij})$



- Valve: pressure decrease $\Delta H_{ij}(t)$, bilinear consistency condition $\Delta H_{ij}Q_{ij} \ge 0$
- Pump: pressure increase $\Delta H_{ij}(t)$, char. diagrams (pressure, flow, speed, power)



Boundary Value Problem

Basic Model (Burgschweiger, Gnädig, St. 2004)

Reservoir j
$$H_j \equiv \bar{H}_j \in C^0(I, \mathbf{R}_{\geq 0})$$
water levelJunction j $\sum_{i: ij \in \mathcal{A}} Q_{ij} - \sum_{k: jk \in \mathcal{A}} Q_{ij} = D_j \in C^0(I, \mathbf{R}_{\geq 0})$ demandTank j $\sum_{i: ij \in \mathcal{A}} Q_{ij} - \sum_{k: jk \in \mathcal{A}} Q_{ij} = A_j(H_j, t)\dot{H}_j, \quad H_j \in C^1(I, \mathbf{R})$ tank inflowPipe ij $H_j - H_i = -\phi_{ij}(Q_{ij}), \quad \phi_{ij} \in C^1(\mathbf{R}, \mathbf{R}) \text{ odd, strictly} \nearrow$ pressure lossPump ij $H_j - H_i = +\Delta H_{ij} \in C^0(I, \mathbf{R}_{\geq 0})$ controlValve ij $H_j - H_i = -\Delta H_{ij} \in C^0(I, \mathbf{R})$ control $Q_{ij}\Delta H_{ij} \geq 0$ consistencyCosts $\int_0^T \sum_{ij \in \mathcal{A} pu} \left[k_{ij}^0 Q_{ij} + k_{ij}^1 P_{ij}(Q_{ij}, \Delta H_{ij}) \right] dt \rightarrow \min$ $k_{ij}^0, k_{ij}^1 \in C^0(I, \mathbf{R}_{\geq 0}), \quad P_{ij} \in C^1(\mathbf{R}^2, \mathbf{R}_{\geq 0})$ power+ initial/final conditions, bounds, inequality constraints

Boundary Value Problem

Basic Model (Burgschweiger, Gnädig, St. 2004)

Reservoir j	$H_{j} \equiv \bar{H}_{j} \in L^{\infty}(I, \mathbf{R}_{\geq 0})$	water level
Junction j	$\sum_{i: ij \in \mathcal{A}} Q_{ij} - \sum_{k: jk \in \mathcal{A}} Q_{ij} = D_j \in L^{\infty}(I, \mathbf{R}_{\geq 0})$	demand
Tank j	$\sum_{i: ij \in \mathcal{A}} Q_{ij} - \sum_{k: jk \in \mathcal{A}} Q_{ij} = A_j(H_j, t) \dot{H}_j, H_j \in H^{1, \infty}(I, \mathbf{R})$	tank inflow
Pipe ij	$H_{j}-H_{i}=-\phi_{ij}(Q_{ij}), \phi_{ij}\in C^{1}(\mathbf{R},\mathbf{R}) \text{ odd, strictly }\nearrow$	pressure loss
Pump ij	$H_{j} - H_{i} = +\Delta H_{ij} \in L^{\infty}(I, \mathbf{R}_{\geq 0})$	control
Valve ij	$H_{j} - H_{i} = -\Delta H_{ij} \in L^{\infty}(I, \mathbf{R})$	control
	$Q_{ij}\Delta H_{ij} \ge 0$	consistency
Costs	$\int_{0}^{T} \sum_{ij \in \mathcal{A}_{pu}} \left[k_{ij}^{0} Q_{ij} + k_{ij}^{1} P_{ij}(Q_{ij}, \Delta H_{ij}) \right] dt \rightarrow \min$	
	$k_{ij}^0, k_{ij}^1 \in L^{\infty}(I, \mathbf{R}_{\geq 0}), P_{ij} \in C^1(\mathbf{R}^2, \mathbf{R}_{\geq 0}) \text{ power}$	
	+ initial/final conditions, bounds, inequality constraints	

Boundary Value Problem

Structure

Semi-explicit DAE-BVP with discontinuities over graph

 $A(x,t)\dot{x} = f(x,z,u,t)$ $\chi = H_{tk}$ 0 = q(x, z, u, t) $z = (Q, H_{rs}, H_{ic})$ $\mathfrak{u} = \Delta \mathsf{H}$

where $A \in L^{\infty}(\mathbb{R}^{n_{\chi}} \times I, \mathbb{R}^{n_{\chi}}_{>0})$, loc. Lipschitz w.r.t. x

Questions

- Index 1? (otherwise stabilization by invariants: Schulz, Bock, St. 1998)
- Consistent initial values?
- Existence/uniqueness of IVP solution?
- Numerical behavior?
- Algebraic structure?
- Special solvers?

Boundary Value Problem

Structure

Semi-explicit DAE-BVP with discontinuities over graph

$$\begin{split} A(x,t)\dot{x} &= f(x,z,u,t) & x = H_{tk} \\ 0 &= g(x,z,u,t) & z = (Q,H_{rs},H_{jc}) \\ \text{where } A \in L^{\infty}(\mathbb{R}^{n_{x}} \times I,\mathbb{R}^{n_{x}}_{>0}), \text{ loc. Lipschitz w.r.t. } x & u = \Delta H \end{split}$$

Answers (St. 2005)

- \checkmark Index 1
- \checkmark Consistent initial values
- $\checkmark~$ Existence/uniqueness of IVP solution
- $\checkmark\,$ Numerical behavior
- ✓ Algebraic structure
- ✓ Special solvers (coarse structure)

DAE Index

Node-Arc Incidence Matrix

$$E(G) \equiv E = \begin{pmatrix} E_{rs} \\ E_{jc} \\ E_{tk} \end{pmatrix} = \begin{pmatrix} E_{pi} & E_{pu} & E_{vl} \end{pmatrix} = \begin{pmatrix} E_{rs,pi} & E_{rs,pu} & E_{rs,vl} \\ E_{jc,pi} & E_{jc,pu} & E_{jc,vl} \\ E_{tk,pi} & E_{tk,pu} & E_{tk,vl} \end{pmatrix}$$

DAE with Incidence Matrix

$$\begin{array}{ccc} \mathcal{A}_{pi} & & \begin{pmatrix} 0 \\ \mathcal{A}_{pu} \\ \mathcal{A}_{pu} \\ \mathcal{A}_{vl} \\ \mathcal{N}_{rs} \\ \mathcal{N}_{rs} \\ \mathcal{N}_{jc} \\ \mathcal{N}_{tk} \end{array} \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ A(t,H_{tk})\dot{H}_{tk} \end{array} \right) = \left(\begin{array}{c} \phi_{pi}(Q_{pi}) + & E_{pi}^{*}H \\ & E_{pu}^{*}H & -\Delta H_{pu} \\ & E_{vl}^{*}H & +\Delta H_{vl} \\ & H_{rs} \\ & H_{rs} \\ & D_{jc} \\ 0 \end{array} \right) - \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ \overline{H}_{rs} \\ D_{jc} \\ 0 \end{array} \right)$$

DAE Index

Control Variables

 $\Delta H_{pu}, \Delta H_{vl}$

Wronskian



DAE Index

Control Variables

 \mathcal{A}_{pu}^{Q} , \mathcal{A}_{vl}^{Q} = flow controlled elements, \mathcal{A}_{pu}^{H} , \mathcal{A}_{vl}^{H} = pressure controlled elements.

Projection Matrices $\Pi_{pu}^{H}, \, \Pi_{pu}^{Q} \text{ on } \mathbf{R}^{|\mathcal{A}_{pu}|} \text{ and } \Pi_{vl}^{H}, \, \Pi_{vl}^{Q} \text{ on } \mathbf{R}^{|\mathcal{A}_{vl}|}$

Wronskian



DAE Index

Control Variables

 \mathcal{A}_{pu}^{Q} , \mathcal{A}_{vl}^{Q} = flow controlled elements, \mathcal{A}_{pu}^{H} , \mathcal{A}_{vl}^{H} = pressure controlled elements.

Projection Matrices $\Pi_{pu}^{H}, \, \Pi_{pu}^{Q} \text{ on } \mathbf{R}^{|\mathcal{A}_{pu}|} \text{ and } \Pi_{vI}^{H}, \, \Pi_{vI}^{Q} \text{ on } \mathbf{R}^{|\mathcal{A}_{vI}|}$

Wronskian constant rank on $\mathbf{R}^{\left|\mathcal{A}_{pi}\right|}$



DAE Index

Theorem

$$v_{\rm D} = v_{\rm P} = v_{\rm T} = 1 \iff \frac{\partial g}{\partial z}$$
 nonsingular

Theorem

Let $\mathcal{A}_{pu}^{H} \cup \mathcal{A}_{vl}^{H}$ pressure-controlled, $G_{0} = G(\mathcal{A}_{pu}^{H} \cup \mathcal{A}_{vl}^{H})$, $\tilde{G}_{0} = G(\mathcal{A}_{pi} \cup \mathcal{A}_{pu}^{H} \cup \mathcal{A}_{vl}^{H})$. The DAE is index-1 \iff

- (a) G_0 is acyclic and contains in every component ≤ 1 element of $N_{rs} \cup N_{tk}$
- (b) \tilde{G}_0 contains \mathcal{N}_{jc} and in every component ≥ 1 element of $\mathcal{N}_{rs} \cup \mathcal{N}_{tk}$

Remarks

- Conditions necessary: physical explanations
- Special cases immediate . . .

Boundary Value Problem

Time Discretization

- Grid $t = 1, 2, \ldots, T$
- Initial values at t = 0
- Variables:

$$\begin{split} \mathbf{x} &= (\mathbf{x}_1, \dots, \mathbf{x}_T) & \mathbf{x}_t &= \mathbf{H}_{tk,t} \\ \mathbf{v} &= (\mathbf{v}_1, \dots, \mathbf{v}_T) & \mathbf{v}_t &= (\mathbf{Q}_t, \mathbf{H}_{rs,t}, \mathbf{H}_{jc,t}, \Delta \mathbf{H}_t) = (\mathbf{u}_t, \mathbf{z}_t) \end{split}$$

Discretized Optimization Problem (NLP)

$$\begin{split} & \underset{(x,\nu)}{\text{Minimize}} \quad \sum_{t=1}^{T} \varphi_t(x_t,\nu_t) = \sum_{t=1}^{T} \sum_{ij \in \mathcal{A}_{pu}} \left[k_{ijt}^0 Q_{ijt} + k_{ijt}^1 P_{ij}(Q_{ijt},\Delta H_{ijt}) \right] \\ & \text{subject to} \quad A_t(x_{t-1},x_t) = f_t(x_t,\nu_t) \quad t = 1,\ldots,T \\ & 0 = g_t(x_t,\nu_t) \quad t = 1,\ldots,T \end{split}$$

+ further restrictions

Boundary Value Problem

KKT System (Newton step for NLP-optimality)

(*)
$$\begin{bmatrix} H^{xx} & H^{xv} & C^{x*} \\ H^{vx} & H^{vv} & C^{v*} \\ C^{x} & C^{v} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta v \\ -\Delta \lambda \end{bmatrix} = \begin{bmatrix} f \\ d \\ h \end{bmatrix}$$

where

$$\begin{split} & \mathsf{H}^{xx} = \mathsf{Diag}(\mathsf{H}^{xx}_t) \\ & \mathsf{H}^{\nu x} = \mathsf{Diag}(\mathsf{H}^{\nu x}_t) \\ & \mathsf{H}^{\nu v} = \mathsf{Diag}(\mathsf{H}^{\nu v}_t) \\ & \mathsf{H}^{\nu v} = \mathsf{Diag}(\mathsf{H}^{\nu v}_t) \end{split} \qquad \mathsf{C}^x = \begin{bmatrix} \mathsf{C}^x_1 \\ \mathsf{F}^x_2 \\ & \mathsf{F}^x_2 \\ & \mathsf{L}^x_3 \\ & \mathsf{C}^x_{\mathsf{T}-1} \\ & & \mathsf{F}^x_{\mathsf{T}-1} \\ & & \mathsf{L}^x_{\mathsf{T}} \\ & & \mathsf{F}^x_{\mathsf{T}} \end{bmatrix} \qquad \mathsf{C}^v = \begin{bmatrix} \mathsf{C}^v_1 \\ & \mathsf{F}^v_1 \\ & & \mathsf{C}^v_1 \\ & & \mathsf{C}^v_{\mathsf{T}} \\ & & \mathsf{F}^v_{\mathsf{T}} \end{bmatrix}$$

Boundary Value Problem

Lemma: DAE is index-1 \iff F_t^{ν} has full row rank

Algorithm: Decoupling of space and time Eliminate control components Δz_t by time-parallel projections (most variables!)

Basically recursion over spanning tree of G \rightarrow local feedback laws $\Delta z_t = M_t \Delta x_t + N_t \Delta u_t + \Delta z_t^0$

Reduced KKT system like (*) where

$$\begin{aligned} & \mathsf{H}^{xx} = \mathsf{Diag}(\mathsf{H}^{xx}_t) \\ & \mathsf{H}^{ux} = \mathsf{Diag}(\mathsf{H}^{ux}_t) \\ & \mathsf{H}^{uu} = \mathsf{Diag}(\mathsf{H}^{uu}_t) \end{aligned} \qquad \mathsf{C}^x = \begin{bmatrix} \mathsf{C}^x_1 & & & \\ \mathsf{L}^x_2 & \ddots & & \\ & \ddots & \mathsf{C}^x_{\mathsf{T}-1} \\ & & \mathsf{L}^x_\mathsf{T} & \mathsf{C}^x_\mathsf{T} \end{bmatrix} \qquad \mathsf{C}^u = \begin{bmatrix} \mathsf{C}^u_1 & & \\ & \ddots & \\ & & \mathsf{C}^u_\mathsf{T} \end{bmatrix} \end{aligned}$$

DP recursion: solve in $O(n^3T)$ where $n = |N_{tk}| + |A_{pu} \cup A_{vl}|$ BWB: $\dim(z_t) = 3346$, n = 70

Discrete Decisions

Types of Discrete Decisions

- Direction of flow in valves harmless
- Switching of pumps
- Alternative outlets

Pump Switching

Dynamic Variables

- Pump $\mathfrak{ij}\in\mathcal{A}_{pu}$
- Flow Q_{ijt}
- Power P_{ijt}
- Pressure increase $\Delta H_{ijt} = H_{jt} H_{it}$

Pump Switching

Dynamic Variables

- Pump $ij \in \mathcal{A}_{pu}$
- Flow Q_{ijt}
- Power P_{ijt}
- Pressure increase $\Delta H_{ijt} = H_{jt} H_{it}$

Aggregation: arcs $ij \in A_{pu}$ actually represent groups of pumps operated in parallel



Pump Switching

Aggregate Variables

- Pump group $ij \in A_{pu}$
- Total flow Q_{ijt}
- Total power P_{ijt}
- Common pressure increase $\Delta H_{ijt} = H_{jt} H_{it}$

Individual Variables

- Pump ijv, $\nu \in \{1,\ldots,N_{ij}\}$
- Flow $Q_{ij\nu t} \in \{0\} \cup [Q_{ij\nu t}^-, Q_{ij\nu t}^+]$: pump status $Y_{ij\nu t} \in \{0, 1\}$
- Speed $n_{ij\nu t}$
- Power $P_{ij\nu t}$

Pump Switching

Aggregate Variables

- Pump group $ij \in \mathcal{A}_{pu}$
- Total flow Q_{ijt}
- Total power P_{ijt}
- Common pressure increase $\Delta H_{ijt} = H_{jt} H_{it}$

Individual Variables

- Pump ij ν , $\nu \in \{1, \dots, N_{ij}\}$
- Flow $Q_{ij\nu t} \in \{0\} \cup [Q_{ij\nu t}^-, Q_{ij\nu t}^+]$: pump status $Y_{ij\nu t} \in \{0, 1\}$
- Speed $n_{ij\nu t}$
- Power $P_{ij\nu t}$

Minimum Up and Down Times (K periods)

$$K(Y_{ij\nu1} - Y_{ij\nu0}) \le Y_{ij\nu1} + \dots + Y_{ij\nu K} \le 2K(Y_{ij\nu1} - Y_{ij\nu0}) + K$$

Pump Aggregation

Essential for Practical Success

- Approximation of hydraulic pressure loss (asymptotically correct, globally smooth)
- Reduction of network graph (parallel pipes, pipe sequences, small subnetworks)
- Initial estimate by Sequential Linear Programming
- Automatic feasibility analysis via penalty approach
- ▷ Pump switching, min up/down times
 - aggregate pump model in network-wide daily planning (NLP model)
 - disaggregate in postprocessing using genuine MINLP per pump group
 - logical constraints by linear model or smooth NCP functions

Pump Aggregation

BWB

- Waterworks have 3 6 parallel outlet pumps
- Model as single aggregated unit with $Q = \sum Q_{\nu} \in \{0\} \cup [\min_{\nu} Q_{\nu}^{-}, \sum_{\nu} Q_{\nu}^{+}]$
- Replace power model + switching with efficiency model

Combined Efficiency of Pump Collection



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Gas Management

Project

Optimization of the Load Distribution



Cooperation: K. Ehrhardt (ZIB), R. Schultz (U Duisburg), A. Martin (TU Darmstadt), Ruhrgas AG (Essen), PSI AG (Berlin) – Support: BMBF

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Gas Management

Gas vs. Water

Differences

- Breathing horizon $\mathsf{T} \in [\mathsf{24}\,\mathrm{h}, \mathsf{48}\,\mathrm{h}]$
- Finer time discretization $\Delta t \in [10 \min, 30 \min]$
- More coarsely meshed networks
- Compressibility
 - PDE for real gas dynamics
 - Pressure and density replace head
 - Extended incidence matrix
 - More complex Wronskian
 - Very ill-conditioned
 - More complex combinatorics (compressor stations, regulators, stop valves)
- \rightarrow Further research needed



Gas Management

Project

Main Results (Ehrhardt, St. 2004/2005, St. 2006)

- Theory
 - Structure of extended incidence matrix
 - Structure of Wronskian (upwind method)
 - Structure of full KKT system
- Algorithmics
 - Control space factorization
 - Locally projecting null space factorization (sparse \checkmark , tree recursion?)
 - Comparison with multifrontal solver MA27 for realistic network (Ruhrgas backbone network)
 - * up to 93% CPU time reduction (factor 13)
 - * up to 75% memory reduction
 - * slightly less accurate

Future

Cooperation with WINGAS? (computation of capacities, operative planning)

Summary

Results

- Complete DAE theory with topological index criteria
- Treat combinatorial aspects by NLP techniques
 - Pump switching by aggregated efficiency model
 - Minimum up/down times by linear constraints and smoothed NCP functions
- Obtain practically satisfactory solutions
- CPU time 15 25 minutes for main network (down from ≈ 1 hour for test configuration!)
- GAMS optimization module at BWB in operation since 2005

Challenges

- Custom NLP solver: work in progress
- Genuine MINLP solver???

Practice: be careful . . .

PWN-proef oorzaak ravage Beverwijk

Waterleidingbedrijf PWN heeft een mislukte proef in Beverwijk, vorig jaar, geheim gehouden. Tot voor kort had de gemeente geen idee wat de oorzaak van de ravage op 19 februari was.

Toen stroomden er miljoenen liters water op straat, sprong een gasleiding en reed een bestelbusje in een gat in de weg. De schade was ruim 100.000 euro.

Recent bleek uit een intern PWN-rapport dat een mislukte proef om een nieuwe computerprogramma te testen, de oorzaak was. Toen monteurs de verkeerde buizen dichtdraaiden, sprongen er verschillende leidingen. Het computerprogramma zou een deel van het personeel overbodig maken.

(from Arie Koster, no source mentioned)