Minisymposium 6 - Positive definite functions and applications

Kernel-based meshless methods for solving PDEs

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This talk provides a framework to derive error bounds and convergence rates for certain unsymmetric meshless methods, including the technique started by E. Kansa in 1986 and the Meshless Local Petrov Galerkin method (MLPG) of S.N. Atluri and collaborators, dating back to 1998. It consists of four essential ingredients:

(1) continuous dependence of the solution of the analytic problem on the data,
(2) a space of *trial* functions allowing a reasonably good approximation to functions in the regularity class of the solution,
(3) a weak or strong *testing* strategy with a certain stability property with respect to the trial space,
(4) a numerical solution of an overdetermined unsymmetric linear system within a certain tolerance.

The theory is not constrained to elliptic problems. It will be shown how to apply the framework for special situations where meshless translates of *kernels* are used as trial functions. In case of weak problems, meshless translates of kernels occur also as test functions, while the test side of strong problems is handled by collocation. Thus our framework covers methods in strong and weak form. In the weak case it allows distributional data, providing error bounds in negative Sobolev norms.